Some Results on the Palette Index of Sierpinski-like Graphs

Department of Informatics and Applied Mathematics,
Yerevan State University, 0025 Yerevan, Armenia,
e-mail: ghazaryan.aghasi@gmail.com

Palette Index

In 2014, Horňák, Kalinowski, Meszka, and Woźniak introduced a new chromatic parameter called the *palette index* of a graph and can be defined as follows: For a given proper edge coloring ϕ of a graph G, we define the *palette* of a vertex $v \in V(G)$ as the set of all colors appearing on edges incident to v. The palette index $\check{s}(G)$ of G is the minimum number of distinct palettes occurring in a proper edge coloring of G.

Palette Index, Examples

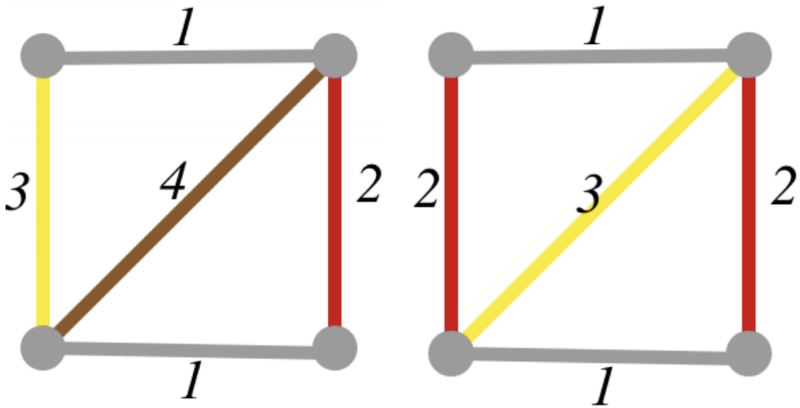

Figure: In the first figure, we used 4 palettes and in the second - 2 palettes

Sierpiński Graph

Sierpiński graphs $S(n, k)$ has been introduced by S. Klavžar, U. Milutinović as a result of the study of the topological properties of the Lipscomb space, and it is still being studied extensively. The graphs are defined as follows. The vertex set of $S(n, k)$ is the all n-tuples of integers $1, 2, ..., k$, namely, $V(S(n, k)) = \{1, 2, ..., k\}^n$. Two different vertices $u = (u_1, ..., u_n)$ and $v = (v_1, ..., v_n)$ are adjacent if and only if there exists an $h \in \{1, ..., n\}$ such that

1. $u_t = v_t$, for $t = 1, ..., h$
2. $u_h \neq v_h$
3. $u_t = v_h$ and $v_t = u_h$ for $t = h + 1, ..., n$.

$S(n, k)$ contains k^{n-1} copies of the graph $S(1, k) = K_k$. We will call *bridge edges* the all edges of $S(n, k)$ those are not in the edges of above mentioned K_k.

Sierpiński Graph, Example

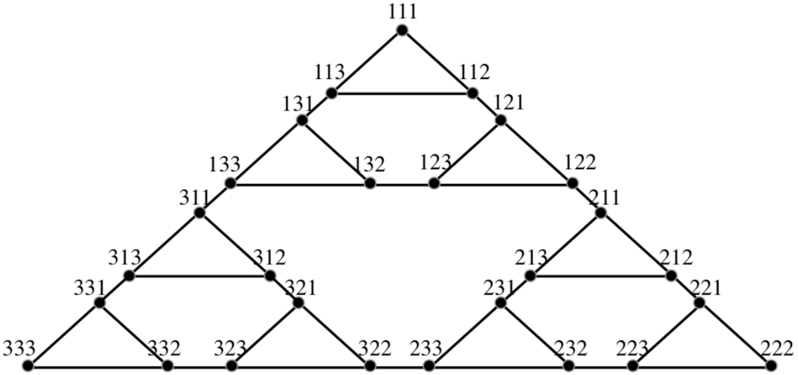

Figure: $S(3,3)$

Sierpiński Gasket Graph

Sierpiński gasket graphs S_n are quite similar to Sierpiński graphs $S(n, 3)$ and are obtained from it by contracting all bridge edges. Sierpiński gasket graphs are fractals and as Sierpiński graphs has many useful properties.

Sierpiński Gasket Graph, Example

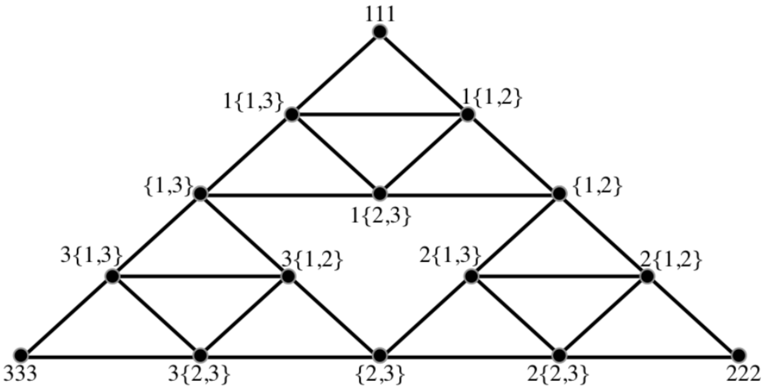

Figure: S_3

Palette Index of Sierpinski Gasket Graph

Theorem

For every positive integer n, the palette index of S_n is determined by this formula:

$$\check{s}(S_n) = \begin{cases} 3 & \textit{if } n \textit{ is odd}, \\ 4 & \textit{if } n \textit{ is even}. \end{cases}$$

Palette Index of Sierpinski Graph

Theorem

For every even integer $k > 1$ *and every integer* $n > 1$, *we have*

$$\check{s}(S(n,k)) = 2.$$

Palette Index of Sierpinski Graph

Proposition

For every odd integer $k > 1$ and every integer $n > 1$, we have

$$\check{s}(S(n,k)) \geq 3$$

Palette Index of Sierpinski Graph

Proposition

For every odd integer $k > 1$ and every integer $n > 1$, we have

$$\check{s}(S(n,k)) \leq \begin{cases} \check{s}(K_k) & \textit{if } n = 2, \\ \check{s}(K_k) + 1 & \textit{if } n > 2. \end{cases}$$

Palette Index of Sierpinski Graph

Corollary

For every integer k, we have

$$\check{s}(S(2, 4k+3)) = 3$$

Palette Index of Sierpinski Graph

Theorem

For every positive integer n, we have

$$\check{s}(S(n,3)) = 3$$

References

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Dziekuje!