Removing induced even cycles from a graph

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University of Birmingham

Joint work with Richard Mycroft

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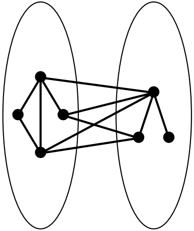

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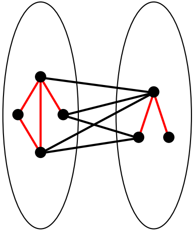

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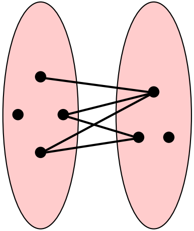

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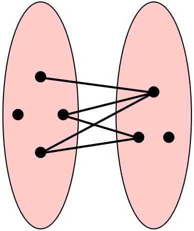

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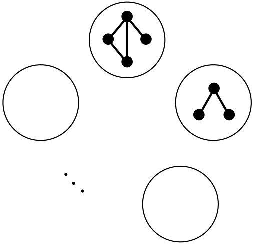

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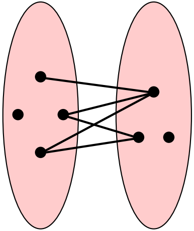

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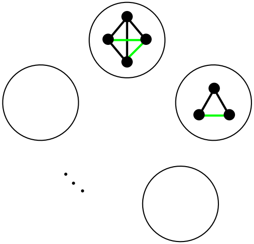

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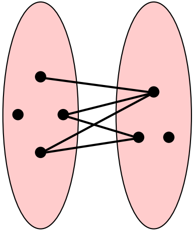

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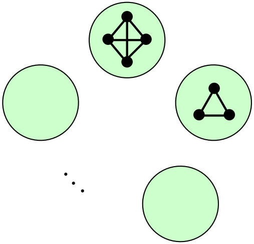

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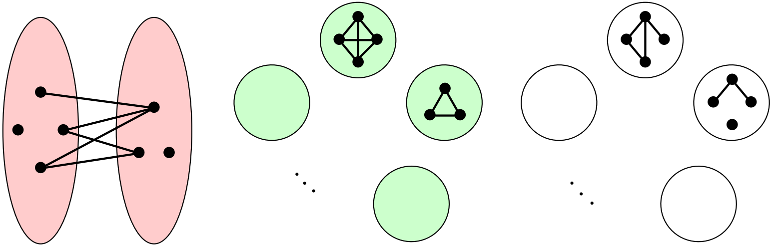

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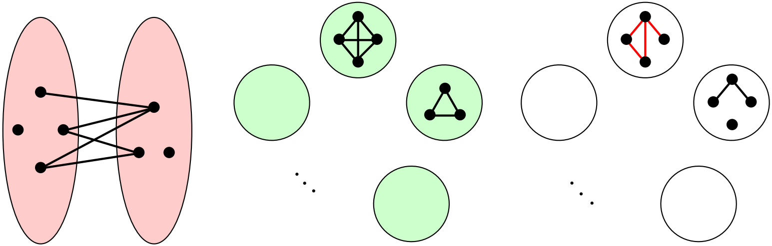

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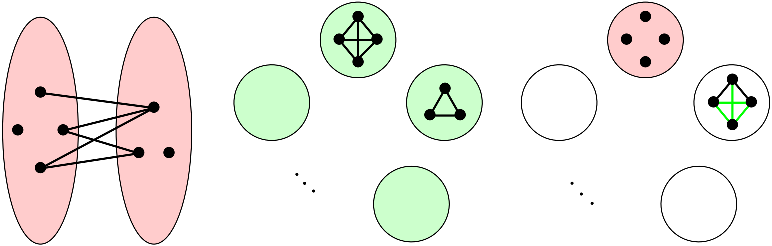

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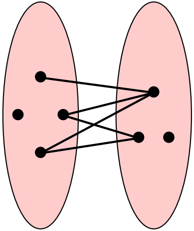

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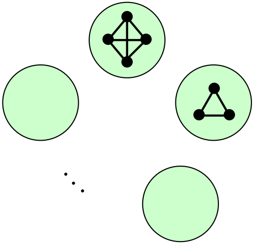

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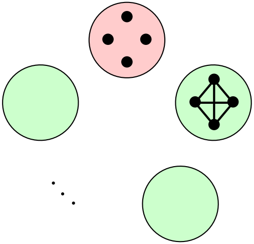

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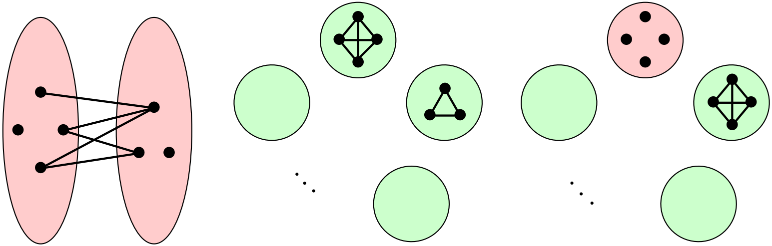

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Goal: minimise the expected proportion of edge changes.

The theory of edit distances - a way to formalise this

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Finding the furthest graph from a hereditary property

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Theorem (Alon and Stav, 2008)

For any hereditary property $\mathcal{H}$, there exists $p^* = p^*(\mathcal{H})$ such that for all n,

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1. The limit exists for all hereditary properties.
2. The function is continuous and concave down.

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If we can find a 'nice' way to calculate $\mathbb{E}\left[\mathrm{dist}(G(n, p), \mathcal{H})\right]$, we are done.

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- If h is odd, then for all $0 \leq p \leq 1$, we have

$$\mathrm{ed}_{\mathrm{Forb}(C_h)}(p) = \min\left\{\frac{p}{2}, \frac{p(1-p)}{1+(\lceil \frac{h}{3} \rceil - 2)p}, \frac{1-p}{\lceil \frac{h}{2} \rceil - 1}\right\}.$$

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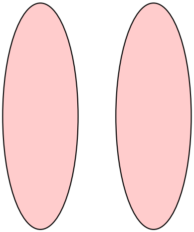

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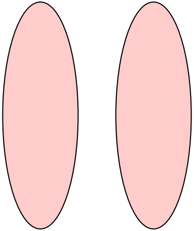

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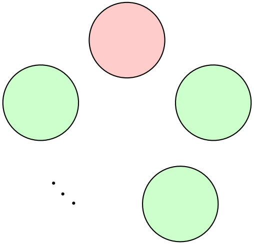

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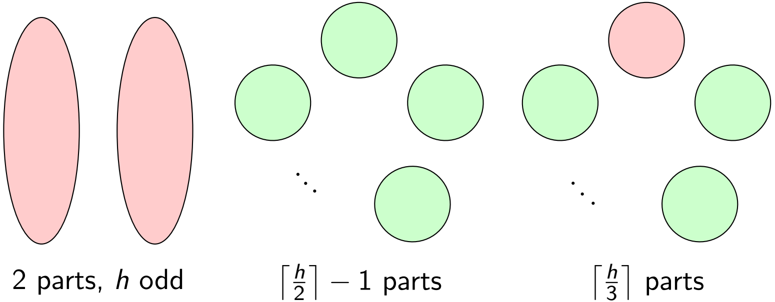

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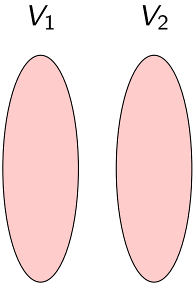

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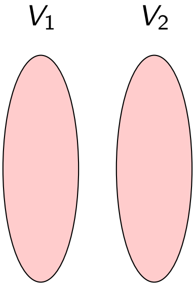

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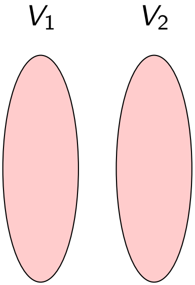

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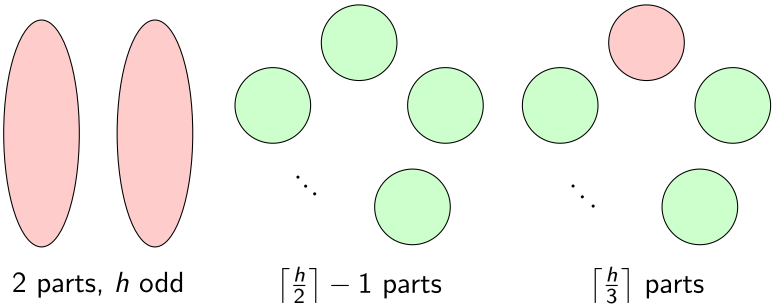

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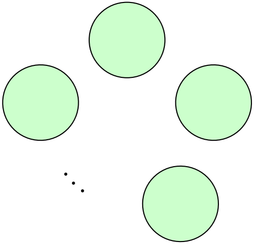

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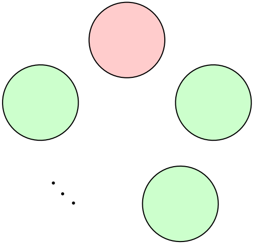

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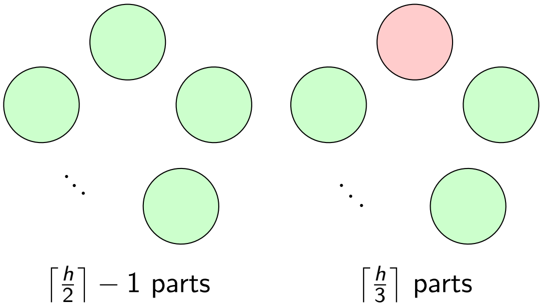

What happens below $p = \lceil h/3 \rceil^{-1}$?

Our result

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If $h \geq 4$ is even, then for all $p_0 \leq p \leq \lceil h/3 \rceil^{-1}$, we have

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In fact, we show a more general result, for the property $\mathrm{Forb}(C_h^t)$.

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Let $h, t \geq 3$ be integers. The t-th power of a cycle C_h, denoted C_h^t, is formed by adding edges between vertices at distance at most t on C_h.

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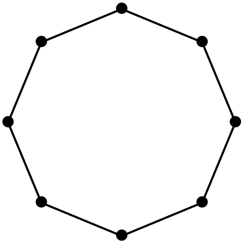

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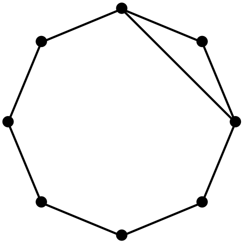

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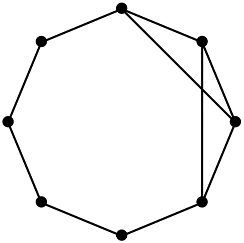

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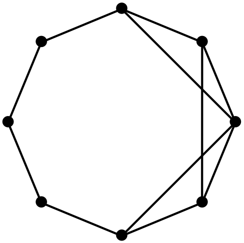

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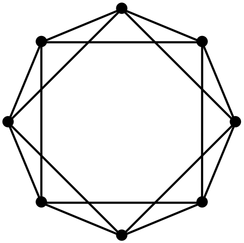

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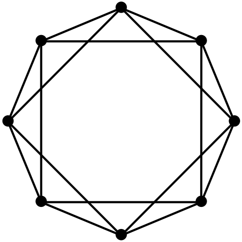

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- When $t = 1$, $p_1 = p_0$ - generalisation of our previous result.

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4. CONTRADICTION!

Thank you!