Group magic labeling of disjoint copies of bipartite graphs

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August 25, 2020

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Magic squares

Definition

A magic square of order n is an $n \times n$ array with entries $1, 2, \ldots, n^2$, each appearing once, such that the sum of each row, column, and both main diagonals is equal to $n(n^2+1)/2$.

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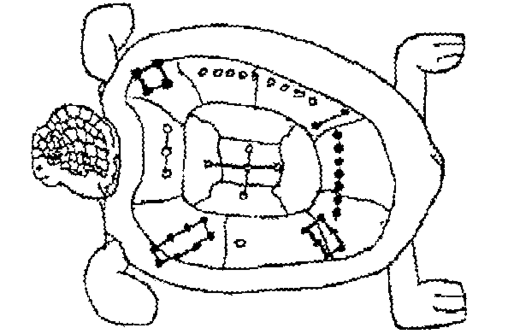
Lo Shu magic square, 2800 B.C



Magic rectangles

Definition

A magic rectangle $MR(a, b)$ *is an* $a \times b$ *array with entries from the set* $\{1, 2, \ldots, ab\}$*, each appearing once, with all its row sums equal to a constant* δ *and with all its column sums equal to a constant* η*.*

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MR(2, 4)

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8	2	3	5

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$\delta = 18$, $\eta = 9$.

Definition

A *magic rectangle* *$MR(a, b)$ is an $a \times b$ array with entries from the set $\{1, 2, \ldots, ab\}$, each appearing once, with all its row sums equal to a constant δ and with all its column sums equal to a constant η.*

MR(2, 4)

1	7	6	4
8	2	3	5

$$\delta = 18,\ \eta = 9.$$

Theorem (Harmuth, 1881)

A magic rectangle $MR(a, b)$ exists if and only if $a, b > 1$, $ab > 4$, and $a \equiv b \pmod 2$.

Magic rectangle set

Definition

A magic rectangle set $M = MRS(a, b; c)$ *is a collection of* c *arrays* $(a \times b)$ *whose entries are elements of* $\{1, 2, \ldots, abc\}$*, each appearing once, with all row sums in every rectangle equal to a constant* δ *and all column sums in every rectangle equal to a constant* η.

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MR(2, 4; 2)

1	15	14	4
16	2	3	13

5	11	10	8
12	6	7	9

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16	2	3	13

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12	6	7	9

$$\delta = 34, \ \eta = 7$$

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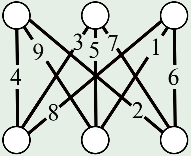
Theorem (Froncek, 2017)

For $a > 1$ *and* $b \geq 4$*, a magic rectangle set* $MRS(a, b; c)$ *exists if and only if* $a, b \equiv 0 \pmod 2$ *or* $abc \equiv 1 \pmod 2$*.*

Magic labeling

Magic labeling

4	9	2
3	5	7
8	1	6



Definition (Sedláček, 1963)

A magic labeling is an injection $f : E \to \mathbb{R}^+ \cup \{0\}$ *such that the sum of the labels of the edges incident to a particular vertex is the same for all vertices.*

Magic labeling

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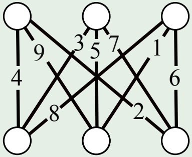

Magic labeling

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A *magic labeling* *is an injection* $f : E \to \mathbb{R}^+ \cup \{0\}$ *such that the sum of the labels of the edges incident to a particular vertex is the same for all vertices.*

Definition (Stewart, 1966)

A magic labeling is *supermagic* *if the set of edge labels consisted of consecutive labels*

Magic labeling

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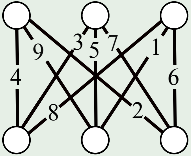
3	5	7
8	1	6



Magic labeling of disjoint copies of bipartite graphs

Definition

For a graph G the union of t disjoint copies of G is denoted by tG.

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Theorem (Shiu, Lam, Cheng, 2000)

Let $t > 1$, $n > 2$. The graph $G = tK_{n,n}$ is supermagic if and only if n is even or both t and n are odd.

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Theorem (Shiu, Lam, Cheng, 2000)

Let $t > 1$, $n > 2$. The graph $G = tK_{n,n}$ is supermagic if and only if n is even or both t and n are odd.

Observe that the existence of a MRS$(n, n; t)$ is **equivalent** to the existence of supermagic labeling of $tK_{n,n}$.

Notation

- Γ - Abelian group, for convenience: 0, $2a$, $-a$, $a-b\dots$
- an involution – an element of Γ of order 2
- $\mathcal{G}$ – the set consisting of all Abelian groups which are of odd order or contain more than one involution

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Γ-magic rectangle set

Definition

A *Γ-magic rectangle set* $MRS_{\Gamma}(a, b; c)$ *on group* Γ *of order* abc *is a collection of* c *arrays* $(a \times b)$ *whose entries are elements of group* Γ*, each appearing once, with all row sums in every rectangle equal to a constant* $\eta \in \Gamma$ *and all column sums in every rectangle equal to a constant* $\delta \in \Gamma$*.*

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$\mathrm{MR}_{\mathbb{Z}_8}(2, 2; 2)$

1	2
6	5

3	0
4	7

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$\mathrm{MR}_{\mathbb{Z}_8}(2,2;2)$

1	2
6	5

3	0
4	7

$\delta = 7,\ \eta = 3$

Results

Theorem (SC,Hinc, 2018+)

Let $\{a, b\} \neq \{2^\alpha, 2l + 1\}$ *for some natural numbers* $\alpha, l > 0$. *A* Γ*-magic rectangle set* $MRS_\Gamma(a, b; c)$ *exists if and only if* a *and* b *are both even or* $\Gamma \in \mathcal{G}$.

Γ-magic labeling

Γ-magic labeling

Definition (Doob, 1974)

Let Γ be an Abelian group. A Γ-magic labeling is an injection $f : E \to \Gamma$ such that the sum of the labels of the edges incident to a particular vertex is the same for all vertices.

Main Results

Theorem (SC, 2020+)

Let $t > 1$. The graph $G = tK_{n,n}$ is Γ-magic for an Abelian group Γ of order tn^2 if and only if $n > 2$ and (n is even or $\Gamma \in \mathcal{G}$).

Theorem (SC, 2020+)

Let $t > 1$. The graph $G = tK_{n,n}$ is Γ-magic for an Abelian group Γ of order tn^2 if and only if $n > 2$ and (n is even or $\Gamma \in \mathcal{G}$).

However in the proof of the above theorem we used a $\mathrm{MRS}_\Gamma(n, n; t)$, observe that the existence of a $\mathrm{MRS}_\Gamma(n, n; t)$ is **not** equivalent to the existence of Γ-magic labeling of $tK_{n,n}$ since there exists a $\mathrm{MRS}_\Gamma(2, 2; t)$ for any Γ of order $4t$.

Thank you