Complexity results on locally-balanced 2-partitions of bipartite graphs

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September 14-18, 2020, 8th Polish Combinatorial Conference

Outline

- Definitions and motivation
- Main results

Definitions and motivation

A 2-*partition* of a graph G is a function

$$f: V(G) \rightarrow \{0,1\}$$

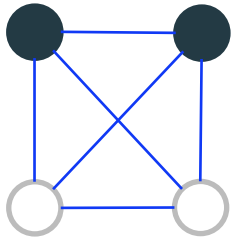
Definitions and motivation

A 2-partition f of a graph G is *locally-balanced with an open neighborhood* if for every $v \in V(G)$,

$$\left| |\{u \in N_G(v): f(u) = 0\}| - |\{u \in N_G(v): f(u) = 1\}| \right| \leq 1,$$

where $N_G(v) = \{u \in V(G): uv \in E(G)\}$.

Example: K_4



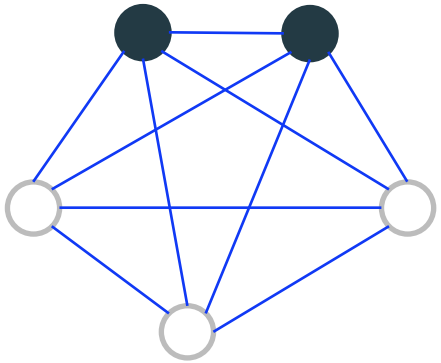
Definitions and motivation

A 2-partition f' of a graph G is *locally-balanced with a closed neighborhood* if for every $v \in V(G)$,

$$\left| |\{u \in N_G[v]: f'(u) = 0\}| - |\{u \in N_G[v]: f'(u) = 1\}| \right| \leq 1,$$

where $N_G[v] = N_G(v) \cup \{v\}$.

Example: K_5



Definitions and motivation

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- The problems of the existence and construction of locally-balanced 2-partitions of graphs are important, since they correspond to the problems concerning a distribution of influences of 2 different powers, which minimizes the probability of conflicts. The subjects of a modelling system may or may not have an ability of self-defense.

Definitions and motivation

- The concept of locally-balanced 2-partition of graphs was introduced by Balikyan and Kamalian in 2005 and it can be considered as a special case of equitable colorings of hypergraphs.
- Berge obtained some sufficient conditions for the existence of equitable colorings of hypergraphs.

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- Kratochvil studied 2-vertex-colorings of graphs for which each vertex is adjacent to the same number of vertices of every color. In particular, he proved that the problem of the existence of such a coloring is NP-complete even for the $(2p, 2q)$-biregular $(p, q \geq 2)$ bipartite graphs. Also, he proved that the problem of the existence of such a coloring could be decided in polynomial time for $(2,2q)$-biregular $(q \geq 1)$ bipartite graphs.

Definitions and motivation

- In 2005, Balikyan and Kamalian proved that the problem of existence of locally-balanced 2-partition with an open neighborhood of bipartite graphs with maximum degree 3 is NP-complete.
  In 2006, they also proved that the problem of existence of locally-balanced 2-partition with an closed neighborhood of bipartite graphs with maximum degree 4 is NP-complete.

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- In 2008, Balikyan obtained the necessary and sufficient conditions for the existence of locally-balanced 2-partitions of trees.

Definitions and motivation

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- In 2016, Gharibyan and Petrosyan obtained the necessary and sufficient conditions for the existence of locally-balanced 2-partitions of complete multipartite graphs.
- In 2017, Gharibyan and Petrosyan derived some results for the existence of locally-balanced 2-partitions of Eulerian graphs.
- In 2018, Gharibyan and Petrosyan obtained some conditions for the existence of locally-balanced 2-partitions of grid-like graphs.

Definitions and motivation

- A bipartite graph is $(\boldsymbol{a}, \boldsymbol{b})$**-biregular** if all vertices in one part have degree a and all vertices in the other part have degree b.
- A graph G is ***even*** if the degree of every vertex of G is even.

Main results

Problem 1.

Instance: A $(3,8)$-biregular bipartite graph G.

Question: Does G has a locally-balanced 2-partition with an open neighbourhood?

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Theorem 1. Problem 1 is NP-complete.

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Problem 2.

Instance: An even bipartite graph G with $\Delta(G) = 4$.

Question: Does G has a locally-balanced 2-partition with an open neighbourhood?

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Instance: An even bipartite graph G with $\Delta(G) = 4$.

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Theorem 2. Problem 2 is NP-complete.

Main results

In 2006, Balikyan and Kamalian proved that the problem of existence of locally-balanced 2-partition with an closed neighborhood of bipartite graphs with maximum degree 4 is NP-complete.

Problem 3.

Instance: A bipartite graph G with $\Delta(G) = 3$.

Question: Does G has a locally-balanced 2-partition with a closed neighbourhood?

Main results

In 2006, Balikyan and Kamalian proved that the problem of existence of locally-balanced 2-partition with an closed neighborhood of bipartite graphs with maximum degree 4 is NP-complete.

Problem 3.

Instance: A bipartite graph G with $\Delta(G) = 3$.

Question: Does G has a locally-balanced 2-partition with a closed neighbourhood?

Theorem 3. Problem 3 is NP-complete.

Main results

Theorem 4. A $(2,2k+1)$-biregular bipartite graph $(k \geq 1)$ has a locally-balanced 2-partition with an open neighbourhood if and only if it has no cycle of length $4r+2$ $(r \geq 1)$.

Dziękuję