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Counting polynomial configurations in subsets of finite fields

A major result in additive combinatorics is Szemerédi theorem, which states that each dense subset of natural numbers contains an arithmetic progression of arbitrary length. From it one can deduce a lower bound for the number of arithmetic progressions of fixed length in subsets of natural numbers. We shall discuss a related question of estimating the number of polynomial progressions in subsets of finite fields. In particular, we estimate the number of configurations of the form

$$x,\ x+y,\ x+y^2,\ x+y+y^2$$

or

$$x,\ x+y,\ x+2y,\ x+y^3,\ x+2y^3$$

in subsets of finite fields.

References

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[2] B. Kuca, *True complexity of polynomial progressions in finite fields*, arXiv:2001.05220.

[3] S. Peluse *On the polynomial Szemerédi theorem in finite fields*, Duke Mathematical Journal, 168, 2019, no. 5, pp. 749-774.