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COUNTING DYCK PATHS WITH COLORED HILLS: A BIJECTIVE APPROACH

It is well known that Dyck paths of semilength n are in bijection with 2-Motzkin paths of length $n - 1$, i.e., both sets are enumerated by the n -th Catalan number C_n . Janjić [1] enumerated Dyck paths of semilength $n - 1$ having colored hills with $m \in \{2, 3, 4\}$ colors. For $m = 2$, he showed that they are also enumerated by C_n , which implies that they are in bijection with 2-Motzkin paths of length $n - 1$. For $m = 3$, he showed that they are enumerated by $\binom{2n-1}{n}$, which implies that they are in bijection with pairs of noncrossing paths [2] of length $n - 1$.

In this work, we present new bijections between Dyck paths with colored hills and various other combinatorial objects, giving bijective proofs for the above results, as well as obtaining some new enumeration results.

This is joint work with Kostas V. Manes.

References

- [1] M. Janjić, *On Enumeration of Dyck Paths with Colored Hills*, Journal of Integer Sequences, 21, 2018, Article 18.9.7
- [2] K. Manes, I. Tasoulas, A. Sapounakis, P. Tsikouras, *Counting pairs of noncrossing binary paths: A bijective approach*, Discrete Mathematics, 342, 2019, pp. 352–359.