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Improved bound on r-distant strong chromatic index

Let $G = (V, E)$ be a simple graph. For a proper edge colouring $c\colon E \to C$ we define *a palette* of a vertex v as

$$S_c(v) = \{c(uv) \mid uv \in E\}.$$

We call c *an adjacent vertex distinguishing edge colouring* if any two adjacent vertices of G have distinct palettes. The least number of colours needed for such colouring is called *the adjacent strong chromatic index* and denoted by $\chi'_a(G)$. This problem was given by Zhang et al. [2] who posed the following conjecture.

Conjecture 1 ([2]) *For every connected graph G other than K_2 or C_5,*

$$\chi'_a(G) \leq \Delta(G) + 2.$$

In this talk we explore a generalization of this problem introduced by Przybyło [1] and inspired by a study of distant chromatic numbers. A proper edge colouring c is *r-adjacent vertex distinguishing* if any two vertices $u, v \in V$, $u \neq v$, such that $d(u, v) \leq r$, receive distinct palettes. The corresponding parameter, denoted by $\chi'_{a,r}(G)$, is *r-adjacent strong chromatic index*.

Przybyło conjectured that there exist constants C and δ_0 such that for any graph G with $\delta(G) \geq \delta_0$, $\chi'_{a,r}(G) \leq \Delta(G) + C$. He confirmed this conjecture for graphs with minimum degree linear to maximum degree. We improve Przybyło's theorem and show that the conjecture holds for any graph with large enough maximum degree.

This is joint work with Magdalena Prorok.

References

[1] J. Przybyło, *Distant set distinguishing edge colourings of graphs*, European Journal of Combinatorics, 69, 2018, pp. 185-199.

[2] Z. Zhang, L. Liu, J. Wang, *Adjacent strong edge coloring of graphs*, Applied Mathematics Letters, 15, 2002, pp. 623–626.