

James Tuite

Open University

TURÁN PROBLEMS FOR k -GEODETIC DIGRAPHS

A Turán-type problem asks for the largest possible size of a graph with given order and a family of forbidden subgraphs \mathcal{F} . Such problems are a foundation of extremal graph theory and have been extensively studied in the setting of undirected graphs; however by comparison Turán problems in directed graphs have received little attention.

The problem for excluded directed cycles and paths has been solved by Bermond et al [1]. We propose the problem of the largest possible size of a k -geodetic digraph with order n . A k -geodetic digraph is a digraph such that, for any two vertices u and v and any directed u, v paths P, Q with length $\leq k$, we have $P = Q$; such digraphs have recently proved to be of great interest in a generalisation of the cage problem. In this presentation we solve this problem and present a partial solution to the more difficult question for the class of strongly-connected digraphs, with conjectured extremal solutions for all k . We conclude with a discussion of generalised Turán problems for k -geodetic digraphs.

This is joint work with Nika Salia, Grahame Erskine, and Olivia Jeans.

References

- [1] J.C. Bermond, A. Germa, M.C. Heydemann, and D. Sotteau *Girth in digraphs*, Journal of Graph Theory, 4(3), 1980, pp. 337–341.