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Modulo 2 and 4 behavior of values of certain partition functions

We introduce two partition functions $f_{a,b}, g_{a,b}$ related to partitions satisfying some non-divisibility conditions. More precisely, $f_{a,b}(n)$ is the number of partitions of n into distinct parts no divisible by a or b; and $g_{a,b}(n)$ is the number of partitions of n into parts not divisible by a or b. We show connection between $f_{2,b}$ and b-regular partitions, and $g_{2,b}$ and partitions with distinct parts not divisible by b. We prove additional results in the case $(a,b) = (2,5)$. We prove certain modulo two congruences for certain subsequences of the sequences $(f_{2,5}(n))_{n \in \mathbb{N}}, (g_{2,5}(n))_{n \in \mathbb{N}}$. Moreover, we obtain the following congruences

$$f_{2,5}(20n + 2) \equiv f_{2,5}(20n + 6) \equiv 0 \pmod{4}.$$

The method used in the proofs allows the proof of a general congruence of the form

$$h_{u,v_1,v_2}(4n + 2) \equiv 0 \pmod{2},$$

where $u = 2s + 1, v_1 = 2r, v_2 = 2r - 2, s \geq r \geq 2$, and $h_{u,v_1,v_2}(n)$ counts the partitions of n into distinct parts which are congruent to $u \pm v_1, u \pm v_2$ (mod $2u$).

This is joint work with Filip Gawron.