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List homomorphism problem parameterized by cutwidth

A homomorphism from a graph G to a graph H is a mapping $\varphi : V(G) \to V(H)$ such that for every edge uv in G it holds that $\varphi(u)\varphi(v) \in E(H)$. For a fixed graph H, in the homomorphism problem, denoted by $\textsc{Hom}(H)$, we are given a graph G and we ask whether there exists a homomorphism from G to H. In the list homomorphism problem, denoted by $\textsc{LHom}(H)$, the graph G is given together with lists L, where for every $v \in V(G)$, the list $L(v)$ is a subset of $V(H)$. We ask whether there exists a homomorphism φ from G to H, which additionally respects the lists, i.e., for every $v \in V(G)$, it holds that $\varphi(v) \in L(v)$. Note that if $H \simeq K_k$, then $\textsc{Hom}(H)$ is equivalent to k-$\textsc{Coloring}$ and $\textsc{LHom}(H)$ is equivalent to $\textsc{List}$-k-$\textsc{Coloring}$.

We study the complexity of $\textsc{LHom}(H)$ parameterized by the cutwidth $\mathrm{ctw}(G)$. Jansen and Nederlof [1] provided an algorithm solving k-$\textsc{Coloring}$ in time $c^{\mathrm{ctw}(G)} \cdot |V(G)|^{\mathcal{O}(1)}$, where c is a constant that does not depend on k. Jansen asked if the same is possible for $\textsc{Hom}(H)$, i.e., if there exists a constant c such that for every H, the $\textsc{Hom}(H)$ problem can be solved in time $c^{\mathrm{ctw}(G)} \cdot |V(G)|^{\mathcal{O}(1)}$. We introduce an invariant, $mim^*(H)$, and we show that for every relevant H, the $\textsc{LHom}(H)$ problem cannot be solved in time $(mim^*(H) - \varepsilon)^{\mathrm{ctw}(G)} \cdot |V(G)|^{\mathcal{O}(1)}$ for any $\varepsilon > 0$, unless the SETH fails. We also provide similar lower bound assuming the ETH, and we use this result to answer the question of Jansen – we prove that there is no constant c such that for every H, the $\textsc{Hom}(H)$ problem can be solved in time $c^{\mathrm{ctw}(G)} \cdot |V(G)|^{\mathcal{O}(1)}$ for every instance G, unless the ETH fails.

This is joint work with Paweł Rzążewski.

References

[1] Bart M. P. Jansen and Jesper Nederlof, *Computing the chromatic number using graph decompositions via matrix rank*, Theoretical Computer Science, 795, 2019, pp. 520–539.