Máté Vizer
Alfréd Rényi Institute of Mathematics

On ordered Ramsey numbers of tripartite 3-uniform hypergraphs

For an integer $k \geq 2$, an *ordered k-uniform hypergraph* $\mathcal{H} = (H, <)$ is a k-uniform hypergraph H together with a fixed linear ordering $<$ of its vertex set. The *ordered Ramsey number* $\overline{R}(\mathcal{H}, \mathcal{G})$ of two ordered k-uniform hypergraphs $\mathcal{H}$ and $\mathcal{G}$ is the smallest $N \in \mathbb{N}$ such that every red-blue coloring of the hyperedges of the ordered complete k-uniform hypergraph $\mathcal{K}_N^{(k)}$ on N vertices contains a blue copy of $\mathcal{H}$ or a red copy of $\mathcal{G}$.

The ordered Ramsey numbers are quite extensively studied for ordered graphs, but little is known about ordered hypergraphs of higher uniformity. We provide some of the first nontrivial estimates on ordered Ramsey numbers of ordered 3-uniform hypergraphs. In particular, we prove that for all $d, n \in \mathbb{N}$ and for every ordered 3-uniform hypergraph $\mathcal{H}$ on n vertices with maximum degree d and with interval chromatic number 3 there is an $\varepsilon = \varepsilon(d) > 0$ such that

$$\overline{R}(\mathcal{H}, \mathcal{H}) \leq 2^{O(n^{2-\varepsilon})}.$$

In fact, we prove this upper bound for the number $\overline{R}(\mathcal{G}, \mathcal{K}_3^{(3)}(n))$, where $\mathcal{G}$ is an ordered 3-uniform hypergraph with n vertices and maximum degree d and $\mathcal{K}_3^{(3)}(n)$ is the ordered complete tripartite hypergraph with consecutive color classes of size n. We show that this bound is not far from the truth by proving $\overline{R}(\mathcal{H}, \mathcal{K}_3^{(3)}(n)) \geq 2^{\Omega(n \log n)}$ for some fixed ordered 3-uniform hypergraph $\mathcal{H}$.

This is joint work with Martin Balko.