Sylwia Cichacz
AGH University of Science and Technology

Group magic labeling of disjoint copies of bipartite graphs

In 1963 Sedláček [3] noticed the following connection between a magic square M of size $n \times n$ and an edge labeling of the complete bipartite graph $K_{n,n}$. Namely, assigning the entry in row i and column j of the magic square to the edge connecting the i-th vertex from the one partite set to the j-th vertex from the second set we obtain the sum of labels around any vertex equals the magic square constant. Stewart calls a magic labeling *supermagic* if the set of edge labels consists of consecutive positive integers [5]. The magic labeling with labels from the elements of an Abelian group has been considered by Doob [1, 2]. A graph G is said to be Γ-*magic* if there is a injection $f\colon E(G) \to \Gamma$ such that for each vertex v the sum of the labels of the edges incident with v are all equal to the same constant, i.e. there exists $\nu \in \Gamma$ such that $\sum_{y \in N(x)} f(x,y) = \nu$ for every $x \in V(G)$.

Shiu, Lam, and Cheng [4] proved that for $n > 2$ a graph $tK_{n,n}$ is supermagic if and only if n is even or both t and n are odd. We obtain a generalization of this theorem for finite Abelian groups. Namely we show that a graph $G = tK_{n,n}$ is Γ-magic for an Abelian group Γ of order tn^2 if and only if $n > 2$ and (n is even or Γ has more than one involution).

References

[1] M. Doob, *Generalizations of magic graphs*, Journal of Combinatorial Theory B, 17, 1974, 205–217.

[2] M. Doob, *Characterizations of regular magic graphs*, Journal of Combinatorial Theory B, 25, 1978 94–104.

[3] J. Sedláček, *Problem 27. In: Theory of Graphs and Its Applications*, M. Fiedler, ed., Praha, 1964, 163–164.

[4] W. C. Shiu, P. C. B. Lam, and H. L. Cheng, *Supermagic labeling of an s-duplicate of $K_{n,n}$*, Congressus Numerantium, 146, 2000, 119–124.

[5] B. M. Stewart, *Supermagic complete graphs*, Canadian Journal of Mathematics, 19, 1967, 427–438.