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Spectra of first-order sentences with quantifier depth 4

We study asymptotic behaviour of the first order properties (properties expressible in first order logic) of binomial random graphs $G(n,p)$. We say that the random graph $G(n,p)$ *obeys the Zero-One k-Law* if for each first-order graph property with quantifier depth no more than k, its probability tends to 0 or tends to 1.

We say that $\alpha \in (0,1)$ *is in k-spectrum* if the random graph $G(n,n^{-\alpha})$ does not obey the Zero-One k-Law. In 1988, it was proven by J. Spencer and S. Shelah that k-spectrum can only contain rational numbers.

In 2012, M. Zhukovskii proved that the smallest number in k-spectrum is $\frac{1}{k-2}$. The full structure of k-spectrum remains unexplained. It is known, however (M. Zhukovskii), that $\frac{1}{2}$ is the limit point of 5-spectrum while 3-spectrum is finite. Finally, it was proven by A. Matushkin and M. Zhukovskii in 2018 that there can be no limit points in 4-spectrum but $\frac{1}{2}$ and $\frac{3}{5}$.

We have tested $\frac{1}{2}$ and $\frac{3}{5}$ on whether they are limit points of 4-spectrum. Thus, we find the minimal k such that k-spectrum is infinite.

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References

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